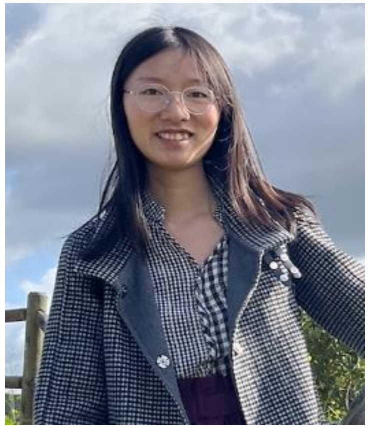
Quantum Metropolis Sampling via Weak Measurement

Jiaqing Jiang (UC Berkeley) and Sandy Irani (UCI)



2025.12.9

Arxiv:2406.16023

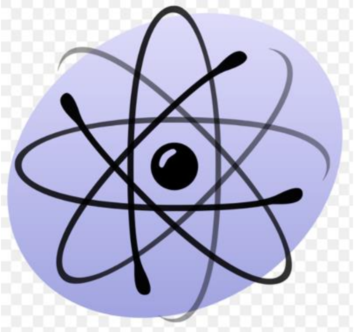
Quantum Gibbs Sampling

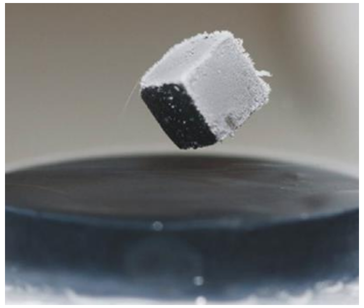
Local Hamiltonian:

$H = H_1 + \ldots + H_m$ Inverse temperature $\beta = 1/T$

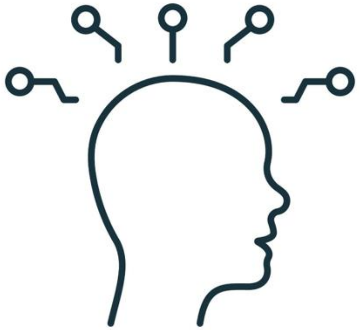
Goal: Given (H, β), prepare Gibbs states $e^{-\beta H}/\mathrm{tr}(e^{-\beta H})$

❖ Distribution over eigenstates





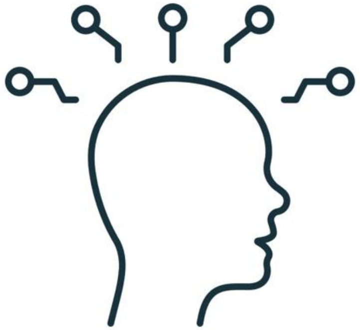
Application of Quantum Gibbs Sampling



$$\rho_\beta \coloneqq e^{-\beta H}/\mathrm{tr}(e^{-\beta H})$$

Quantum Boltzmann Machine

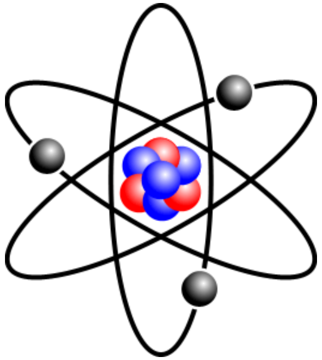
Application of Quantum Gibbs Sampling



$$\rho_\beta \coloneqq e^{-\beta H}/\mathrm{tr}(e^{-\beta H})$$

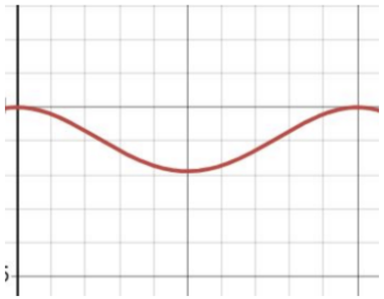
Quantum Boltzmann Machine

- compute the gradient $\mathrm{tr}(H\rho_\beta)$
- Sampling task



Quantum Chemistry

2D Hubbard model



Optimization

Simulated annealing

Quantum Gibbs sampling

Goal: given (H, β), prepare Gibbs states $$\frac{\exp(-\beta H)}{tr(\exp(-\beta H))}$$

Today's focus

Correctness

Converge in **finite** time

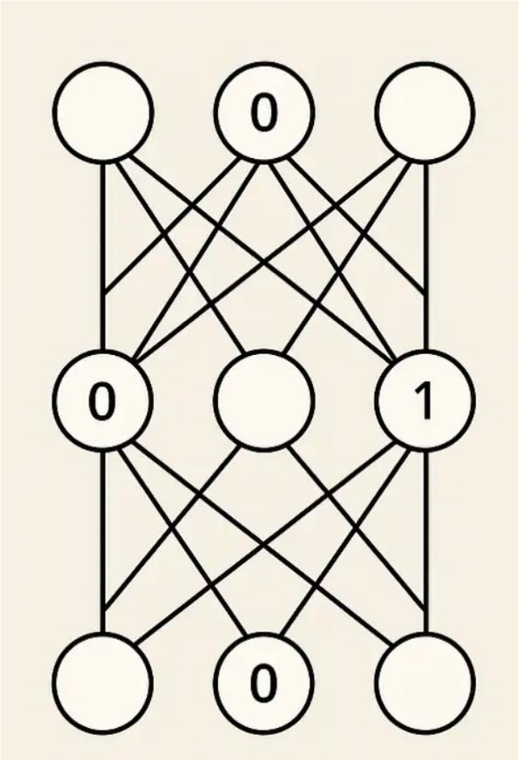
Efficiency

Converge in **poly(n)** time

Outline

- Classical Metropolis Algorithm
- Quantum Metropolis Algorithm & Challenge
- Our algorithm
- Comparison Previous work

Classical Gibbs sampling



Classical Boltzmann machine

$$H = \sum_{<i,j>\in G} w_{ij} Z_i Z_j$$

Goal: Generate the Gibbs state (distribution)

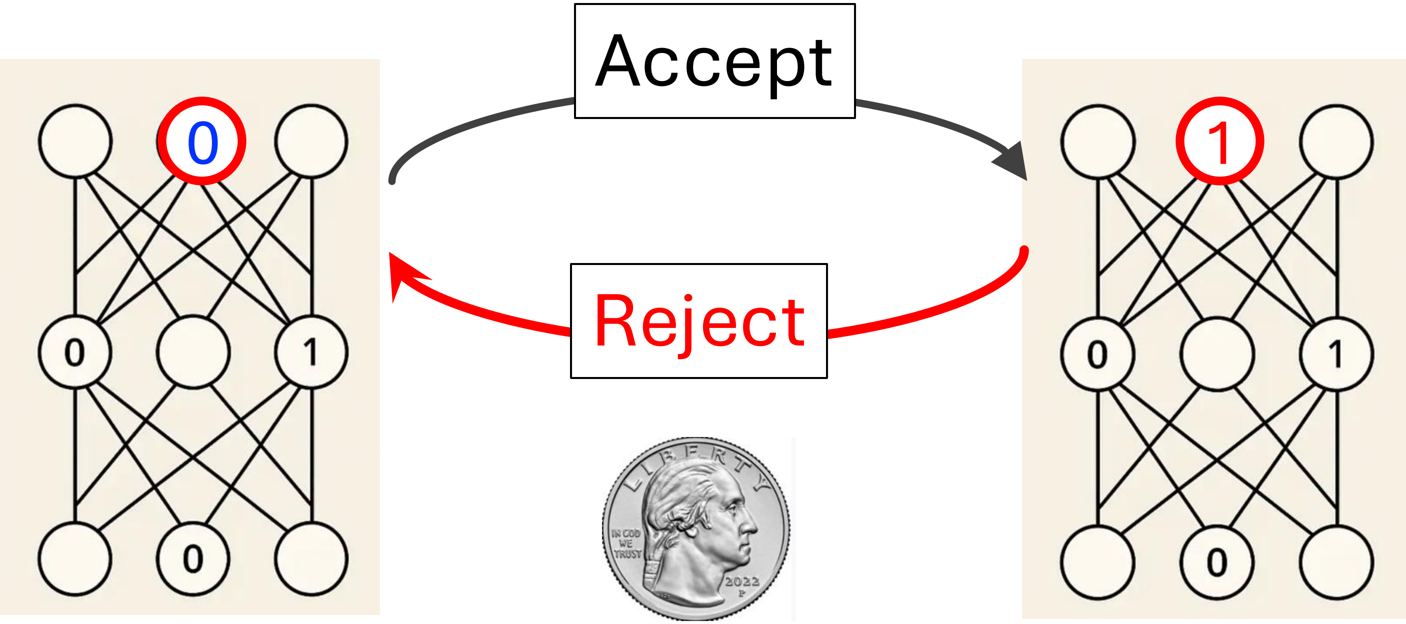
$$01011 \sim \exp(-H(01011))$$

Classical Metropolis Algorithm 1953

Markov chain ⟶ Gibbs distribution

Classical Metropolis Algorithm 1953

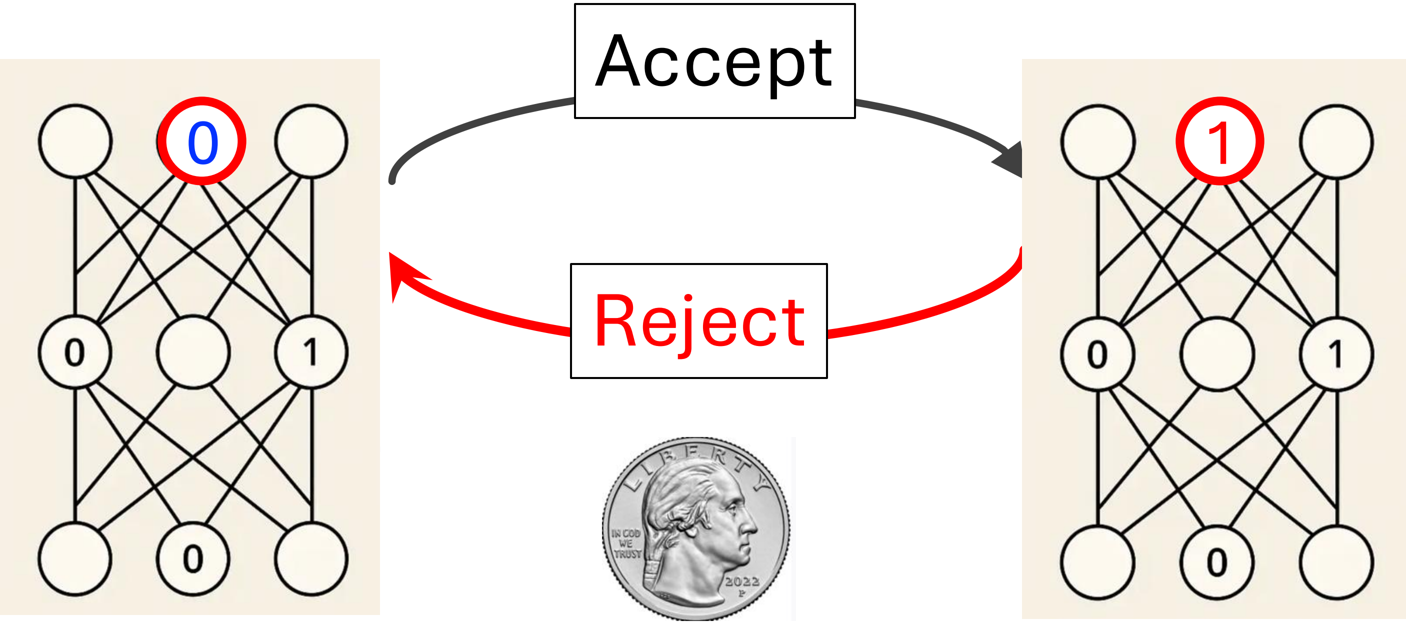
Markov chain ⟶ Gibbs distribution



1) Choose a random vertex and flip it
2) Flip a coin and choose **Accept/Reject**

Classical Metropolis Algorithm 1953

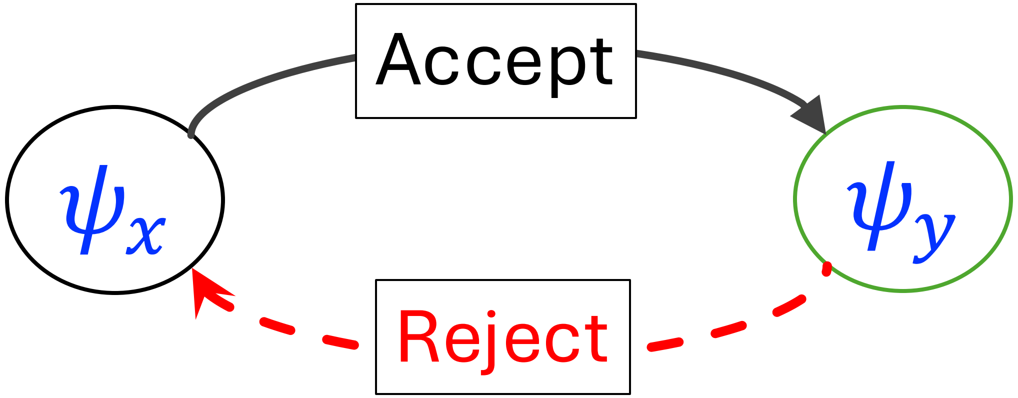
Markov chain ⟶ Gibbs distribution



1) Choose a random vertex and flip it
2) Flip a coin and choose **Accept/Reject**

$$P(\text{head}) = \min\{1, \exp(-\beta\Delta_H)\}$$

Quantum Metropolis: challenges

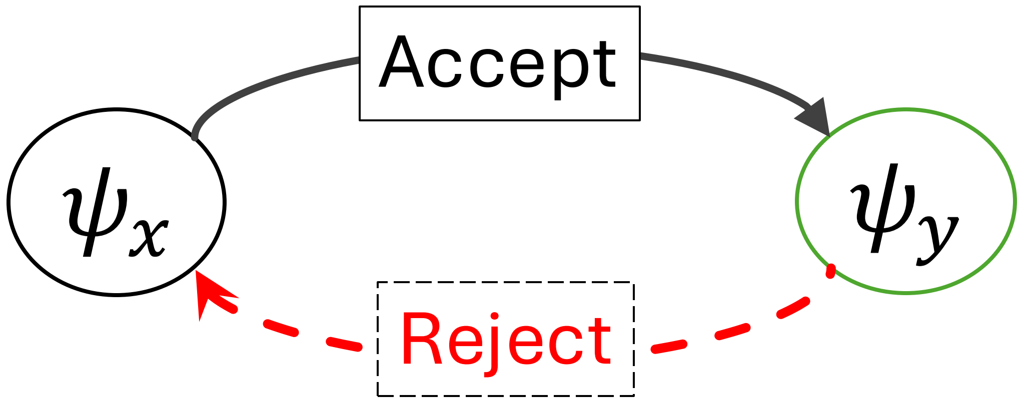


- **Challenge**: Quantum state is **unclonable**
- **Revert** quantum state **after measurement**?

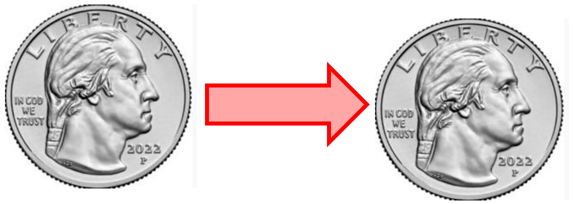
Ideas: **Weak measurement** is easier to revert!
Alternate Accept

Our ideas to revert measurement

1) Weak measurement
2) Alternate Accept



Weak measurement

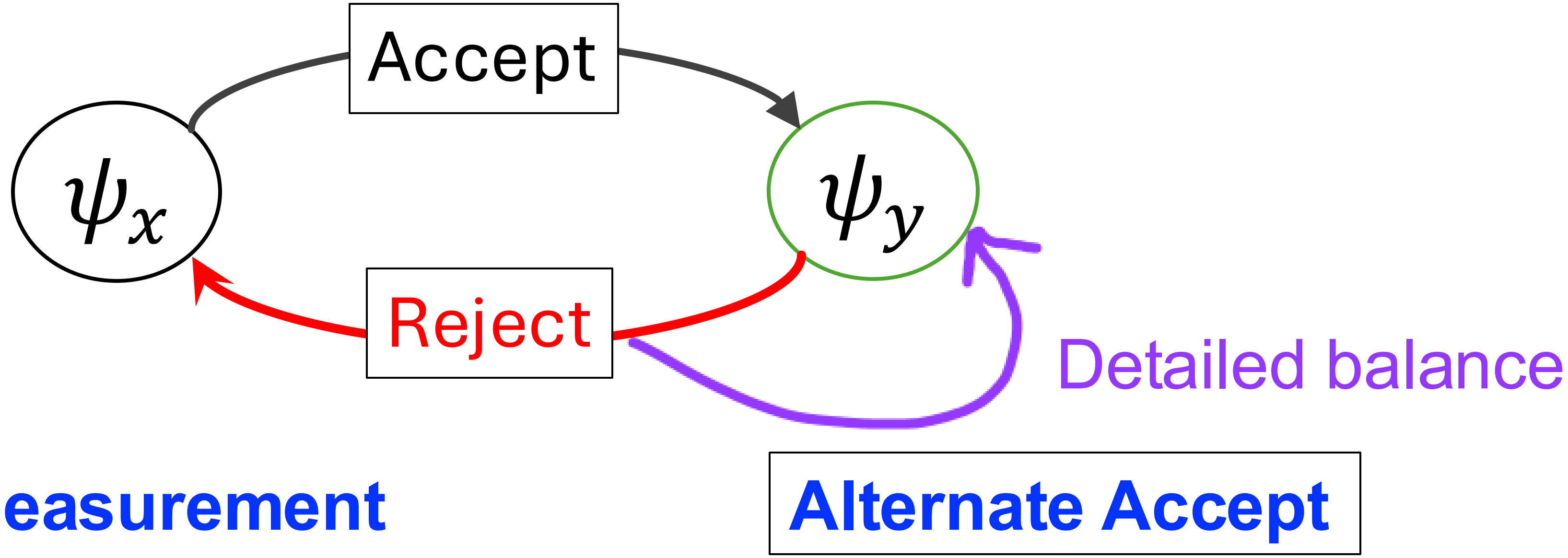


$\times \tau^2$

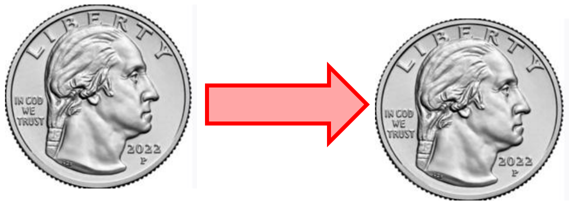
$$\tau = 1/poly$$

Our ideas to revert measurement

1) Weak measurement
2) Alternate Accept



Weak measurement



$\times \boldsymbol{\tau}^2$

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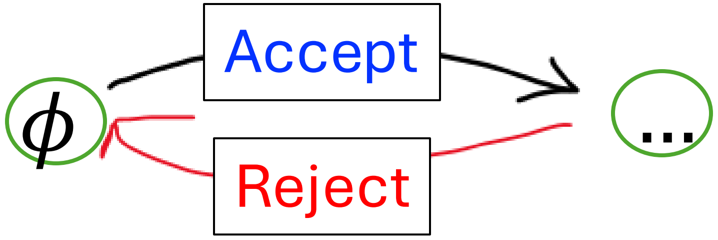
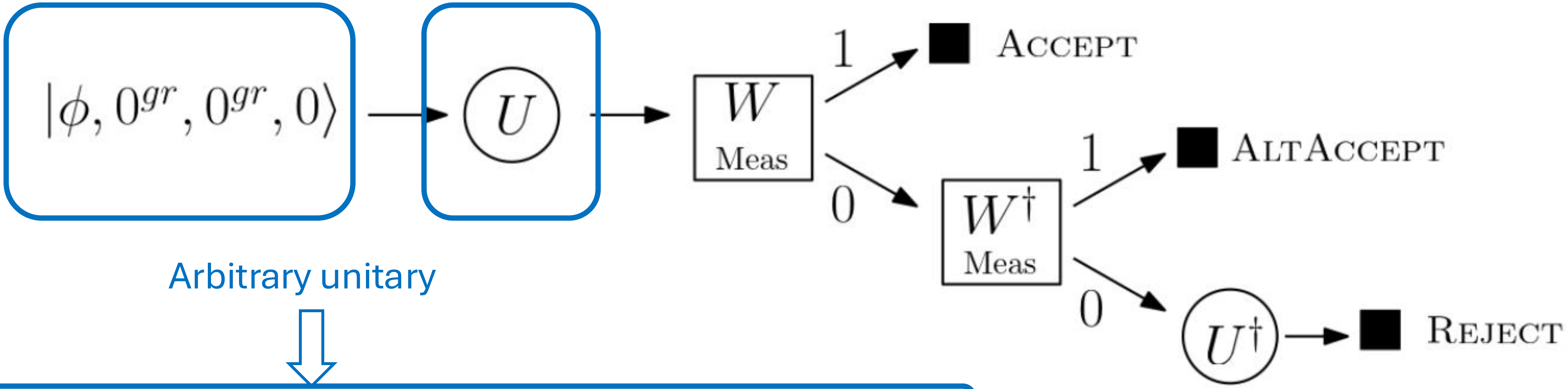


Our algorithm in details

Do something random, then reject with some probability.



Arbitrary unitary

U is $\mathrm{QPE}_{1,3} \circ \mathrm{C} \circ \mathrm{QPE}_{1,2}$.

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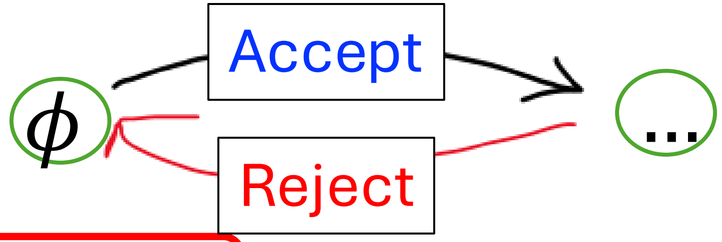

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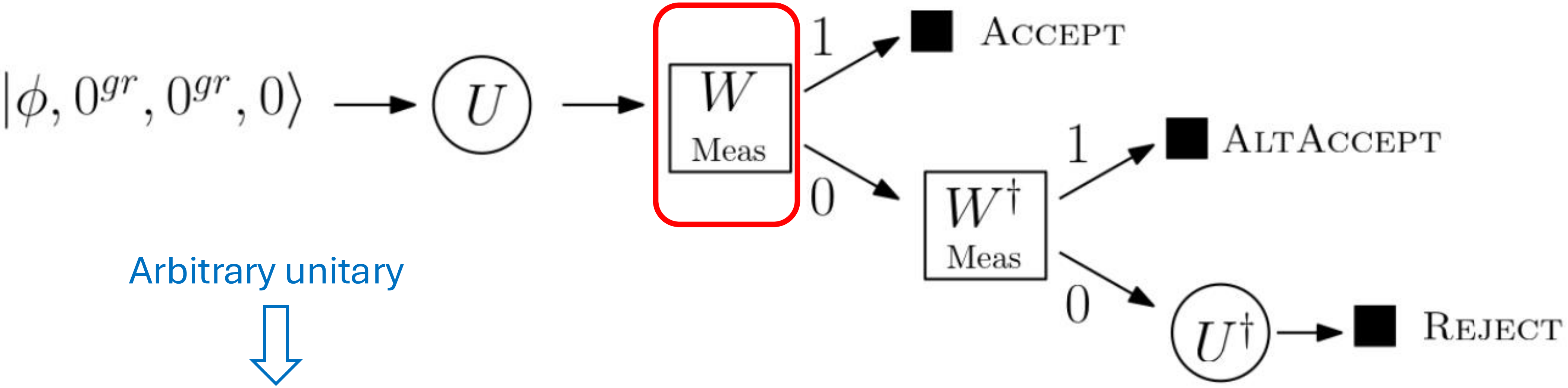

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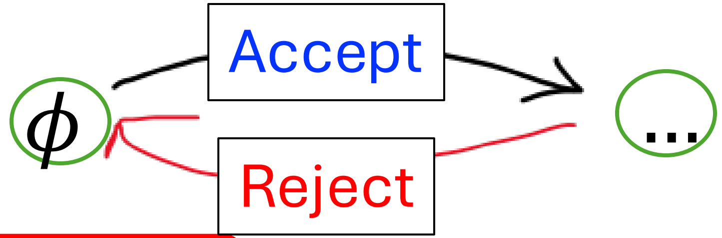
$$W\,|EE'\rangle\,|0\rangle = |EE'\rangle\left(\sqrt{1-\tau^2 f_{EE'}}\,|0\rangle + \tau\sqrt{f_{EE'}}\,|1\rangle\right)$$

Weak Measurement

Our algorithm in details



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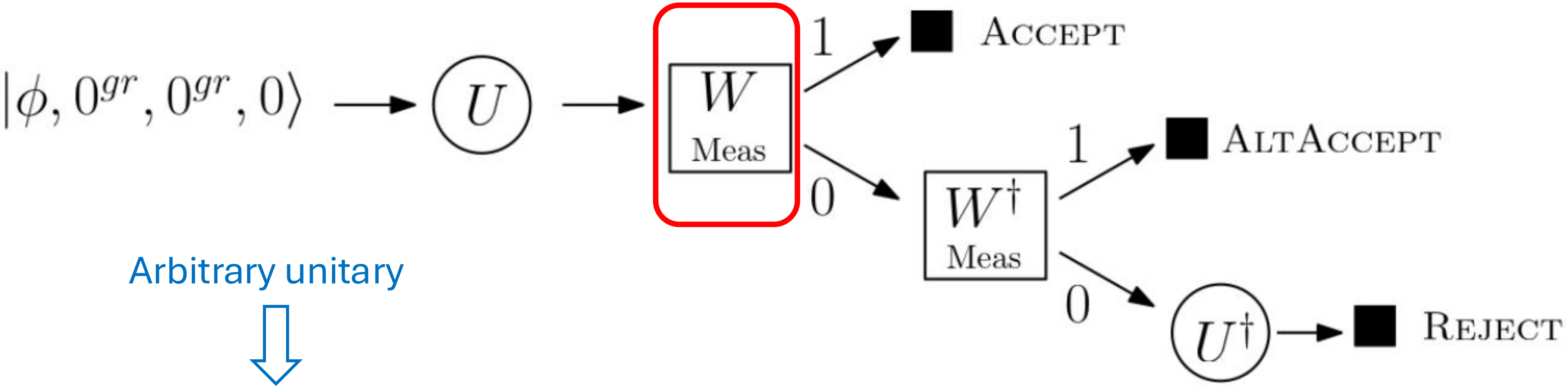

Arbitrary unitary

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$$W |EE'\rangle |0\rangle = |EE'\rangle \left(\sqrt{1 - \tau^2 f_{EE'}}\, |0\rangle + \tau \sqrt{f_{EE'}}\, |1\rangle \right)$$

$$W := \sum_{E,E' \in S(r)^{\otimes g}} |EE'\rangle \langle EE'| \otimes \begin{bmatrix} \sqrt{1 - \tau^2 f_{EE'}} & \tau \sqrt{f_{EE'}} \\ \tau \sqrt{f_{EE'}} & -\sqrt{1 - \tau^2 f_{EE'}} \end{bmatrix}.$$

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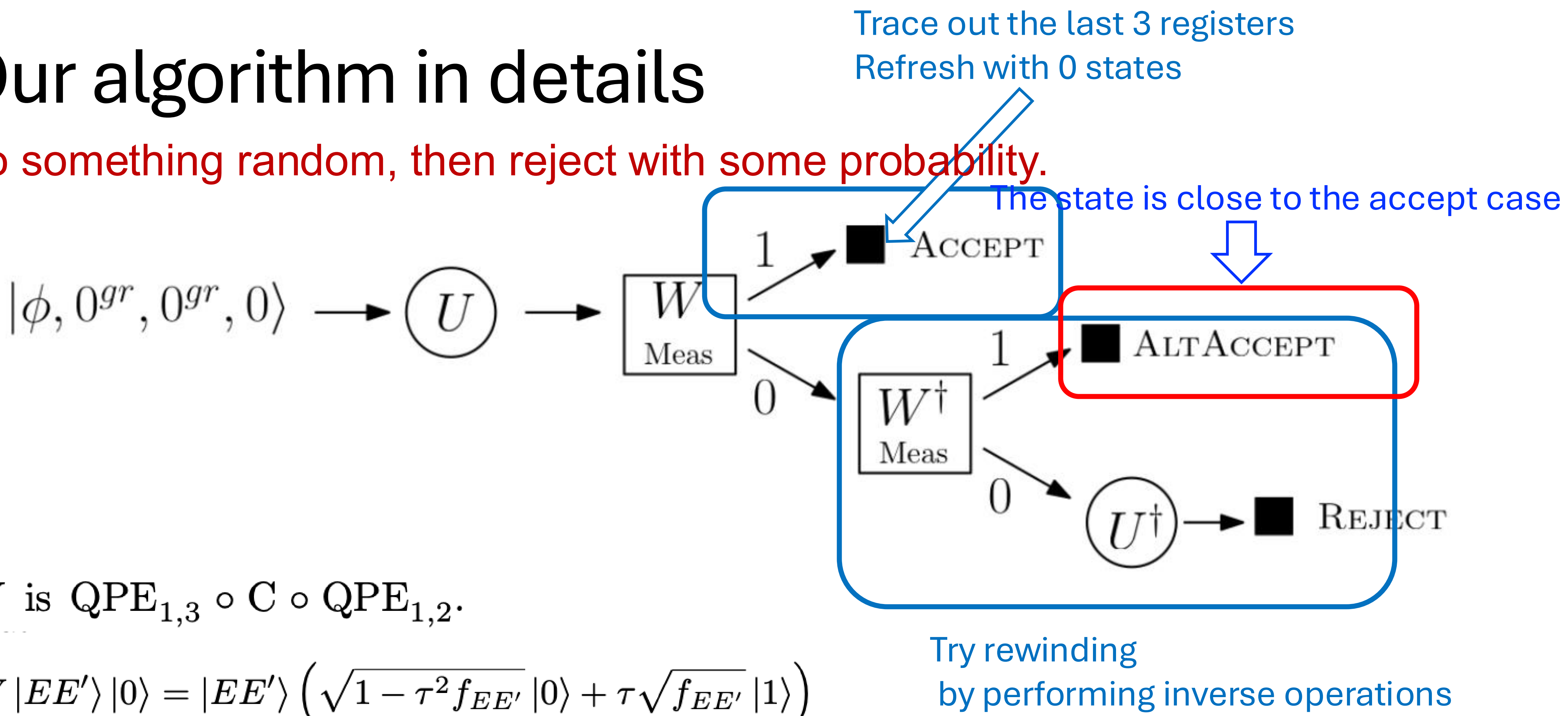

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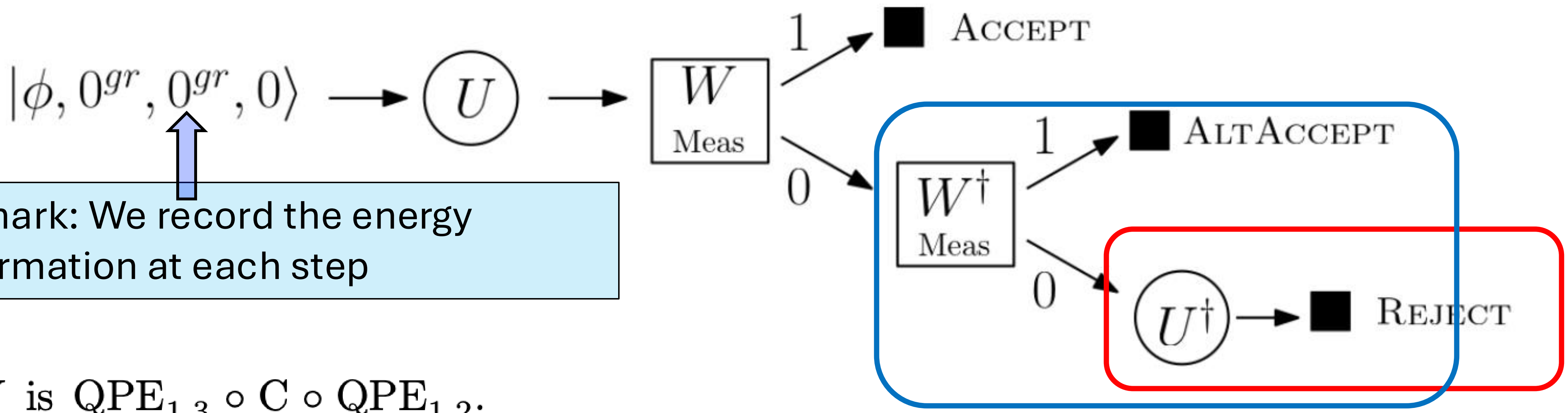


Remark: We record the energy information at each step

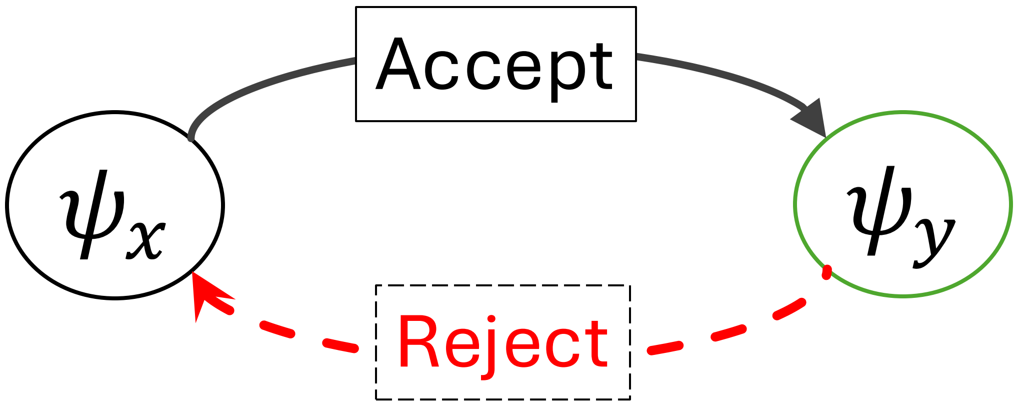
U is $\mathrm{QPE}_{1,3} \circ \mathrm{C} \circ \mathrm{QPE}_{1,2}$.

$$W|EE'\rangle|0\rangle = |EE'\rangle\left(\sqrt{1-\tau^2 f_{EE'}}|0\rangle + \tau\sqrt{f_{EE'}}|1\rangle\right)$$

Try rewinding by performing inverse operations

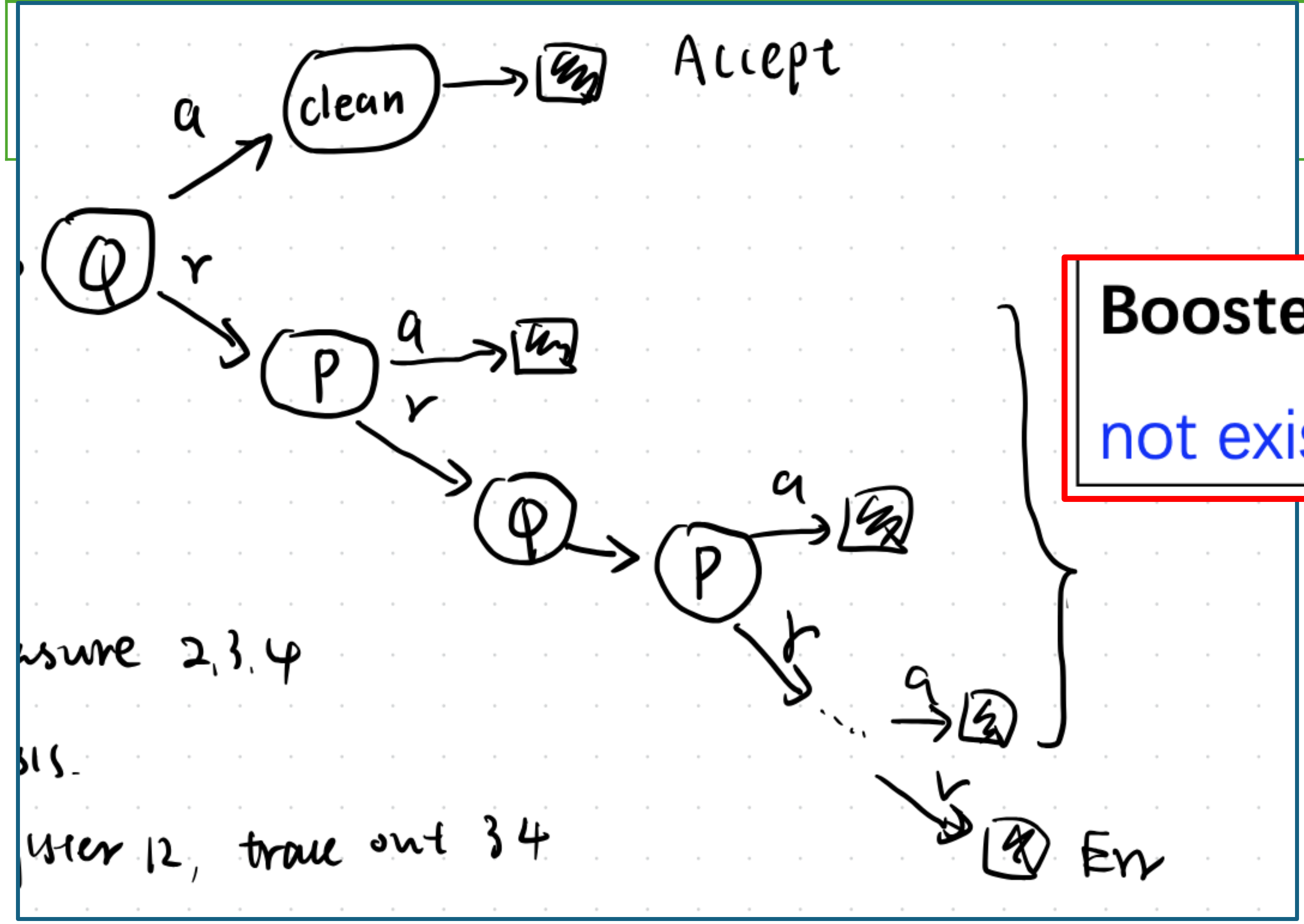
Comparison to existing work

- **Revert** quantum state **after measurement**?



Comparison to existing work

[TOV+11]	Marriot-Watrous rewinding	Metropolis-type, errors in proof



Boosted + **Shift-invariant** QPE does not exist. [CKBG23]

Comparison to existing work

[TOV+11]	Marriot-Watrous rewinding	Metropolis-type, errors in proof
[CKBG23,DLL24]	Davies generator + Operator Fourier transform	provably-correct, differ from Metropolis

$$\mathcal{L}_\beta[\cdot] := \underbrace{-\mathrm{i}[\boldsymbol{B}, \cdot]}_{\text{“coherent”}} + \sum_{a\in A} \int_{-\infty}^{\infty} \gamma(\omega) \left(\underbrace{\hat{\boldsymbol{A}}^a(\omega)(\cdot)\hat{\boldsymbol{A}}^a(\omega)^\dagger}_{\text{“transition”}} - \underbrace{\frac{1}{2}\{\hat{\boldsymbol{A}}^a(\omega)^\dagger \hat{\boldsymbol{A}}^a(\omega), \cdot\}}_{\text{“decay”}} \right) \mathrm{d}\omega$$

$$\hat{\boldsymbol{A}}^a(\omega) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\boldsymbol{H}t} \boldsymbol{A}^a \mathrm{e}^{-\mathrm{i}\boldsymbol{H}t} \mathrm{e}^{-\mathrm{i}\omega t} f(t) \mathrm{d}t$$

$$\gamma(\omega) = \exp\left(-\frac{(\omega + \omega_\gamma)^2}{2\sigma_\gamma^2} \right)$$

Comparison to existing work

[TOV+11]	Marriot-Watrous rewinding	Metropolis-type, errors in proof
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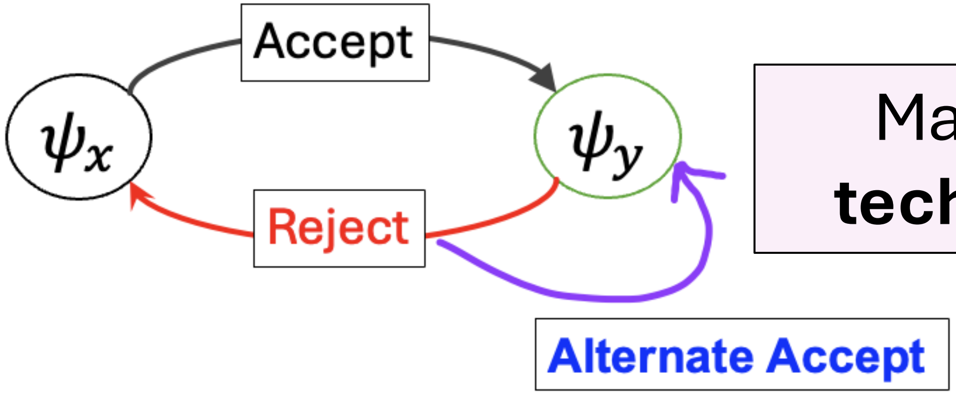
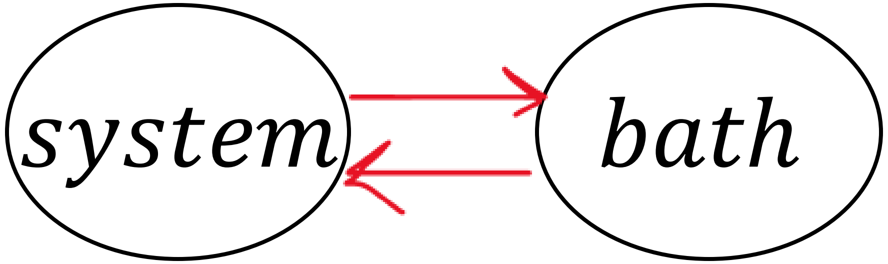
Our work [JI24]	**Weak measurement + Alternate accept**	**conceptually-simple** and **provably correct**



Make it easier to **generalize classical techniques to quantum** Gibbs sampling

Comparison to existing work

[TOV+11]	Marriot-Watrous rewinding	Metropolis-type, errors in proof
[CKBG23,DLL24]	Davies generator + Operator Fourier transform	provably-correct, differ from Metropolis
Our work [JI24]	**Weak measurement + Alternate accept**	**conceptually-simple** and **provably correct**
[DZPL25,HW25,HPB25, LA25…]	… System bath interactions…	conceptually simple and provably correct



$$H_\alpha(t) = H + H_E + \alpha f(t) \left(A_S \otimes B_E + A_S^\dagger \otimes B_E^\dagger \right).$$

In summary….

Most existing work

Correctness

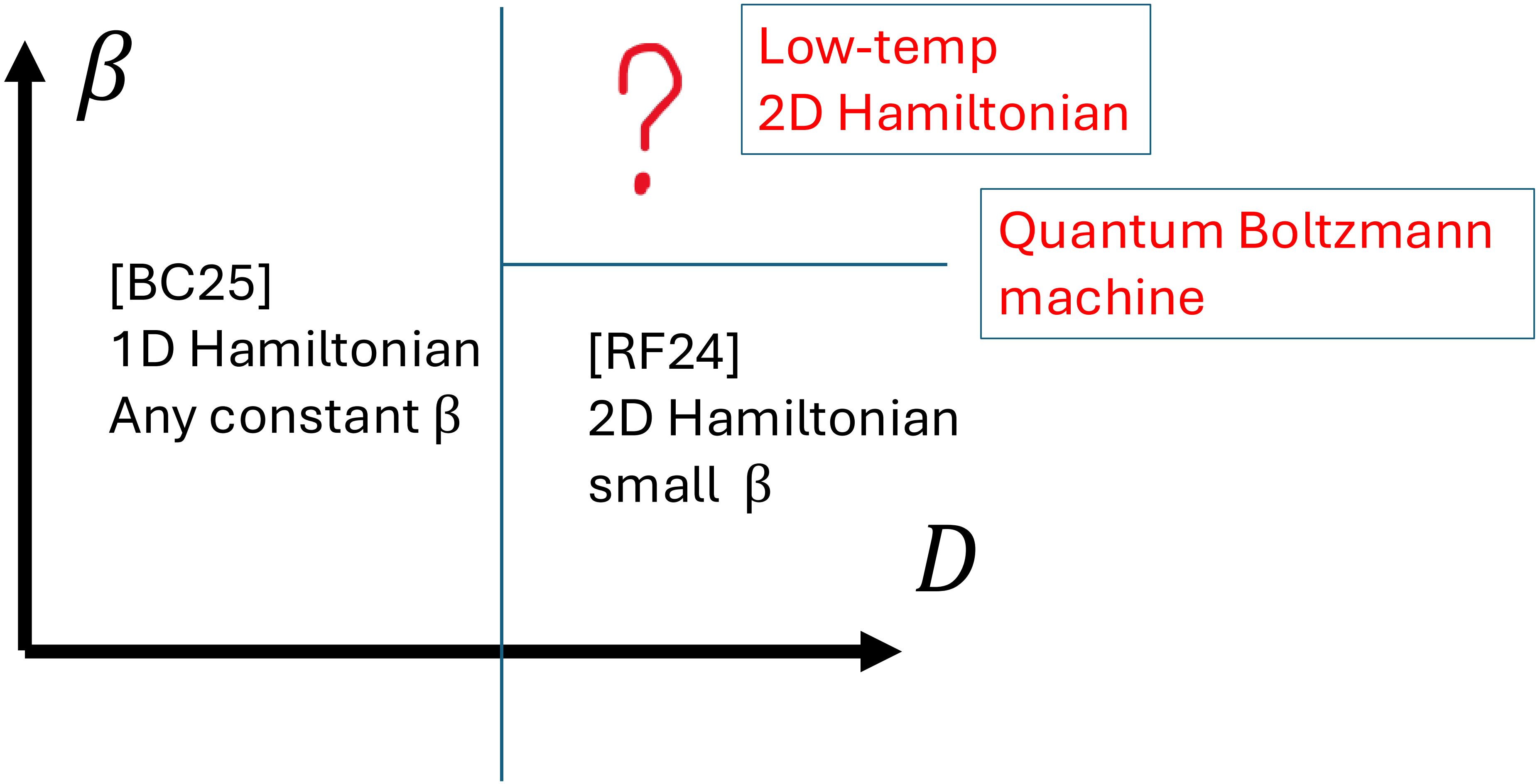
Converge in **finite** time

Future direction

Efficiency

Converge in **poly(n)** time

Q1: When quantum Gibbs sampling* is efficient?



Q2: When estimating $tr(O\rho_\beta)$ is efficient?

- [?] Design an algorithm that requiring $tr(O\rho_t) \to tr(O\rho_\beta)$

  might be much easier than requiring $\rho_t \to \rho_\beta$

Q2: When estimating $tr(O\rho_\beta)$ is efficient?

- [?] Design an algorithm that requiring $tr(O\rho_t) \to tr(O\rho_\beta)$
  might be much easier than requiring $\rho_t \to \rho_\beta$

- Get effective independent sample of ρ_β with a time smaller than t_{mix} [incoming work, IPAM workshop at UCLA, 2026.1.12-202.1.16]

New Frontiers in Quantum Algorithms for Open Quantum Systems

[JI24] might make it easier to **generalize classical techniques to quantum** Gibbs sampling

Thanks for listening. Questions ?

Appendix